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Abstract

Analogical reasoning, the ability to understand phenomena as systems of structured relationships that can be aligned, compared, and mapped together, plays a fundamental role in the technology rich, increasingly globalized educational climate of the 21st century. Flexible, conceptual thinking is prioritized in this view of education, and schools are emphasizing "higher order thinking," rather than memorization of a cannon of key topics. The lack of a cognitively grounded definition for higher order thinking, however, has led to a field of research and practice with little coherence across domains or connection to the large body of cognitive science research on thinking. We review literature on analogy and disciplinary higher order thinking to propose that relational reasoning can be productively considered the cognitive underpinning of higher order thinking. We highlight the utility of this framework for developing insights into practice through a review of mathematics, science, and history educational contexts. In these disciplines, analogy is essential to developing expert-like disciplinary knowledge in which concepts are understood to be systems of relationships that can be connected and flexibly manipulated. At the same time, analogies in education require explicit support to ensure that learners notice the relevance of relational thinking, have adequate processing resources available to do so, and align structures of relationships with critical attention to limits to the alignments between analogs.

Keywords: analogy, higher order thinking, representations, mathematics education, science education, history education

Shifts in technology, the ubiquity of the internet, the global market economy, and broadening community commitments to formal education worldwide have altered the goals and functions of formal education.[1; 2] A growing body of theory across fields has begun to delineate a series of skills – often called higher order thinking – that are distinct from the traditional academic cannons of facts (e.g. mathematics, science, history) and that may be more predictive of high quality educational and employment outcomes in the current market economy.[1-3] With much information readily available through ubiquitous computing and search engines, cognitive skills that support the capacity to categorize, generalize, draw inferences from, and otherwise transform knowledge may be more foundational to successful academic performance as well as to longer-term societal and economic participation.

Analogical reasoning is one such cognitive skill that underpins many of these 21st Century competencies. Analogical reasoning is the process of representing information and objects in the world as systems of relationships, such that these systems of relationships can be compared, contrasted, and combined in novel ways depending on contextual goals.[4-5,] This manuscript explores how analogical reasoning serves as a unifying mechanism underlying higher order thinking skills, both as a tool for promoting content acquisition and as a basic cognitive mechanism for using information flexibly and across contexts. National Research Council reports have proposed that transforming academic content and transferring from one context to another are key skills to expert-like academic competency as well as to more broadly applicable employment opportunities in the 21st century[1,6]. Similarly, academic disciplinary experts have echoed this call for greater focus on higher order thinking, transfer, and generalization of knowledge, including in mathematics, [6-8] science,[9] and history.[10] How to support students in developing these skills is less clear, since these are ill-defined terms and practices, but identifying analogy as an underpinning of higher order thinking enables cognitive science to bear on these educational debates. The relationship between historical and contemporary approaches to defining higher order thinking and the cognitive processing of relational reasoning are discussed next, followed by a treatment of disciplinary higher order thinking in mathematics, science, and history.

THE COGNITIVE SCIENCE OF HIGHER ORDER THINKING

There is consensus that 21st century education should prioritize students' skills for higher order thinking, transfer and flexible reasoning over memorization of disciplinary facts, though what that means in practice is less clear. Part of the challenge is that there is little agreement regarding an operational definition for "higher order thinking."[11] Existing models of higher order thinking are briefly reviewed, and then a new structure-mapping model of higher order thinking is proposed as a way to develop an implementable model that would enable interconnections between the educational and cognitive psychological literatures.

Models of Higher Order Thinking

Higher order thinking as a term has been used broadly and defined many times across disciplines and within content domains. For example, within philosophy, Meier[12-13] proposed an early formulation that distinguished between levels of reasoning based on whether the task involved productive thinking, or reasoning, versus learned, or re-productive thinking. The former might be described as higher order thinking if it involved drawing on prior knowledge to make inferences and solve problems. Bartlett[14] alternatively described higher order thinking as drawing on interpolation, extrapolation, and reinterpretation processes to fill gaps in one's prior knowledge. Following a review of these historical treatments of higher order thinking within philosophy, Lewis and Smith suggest that: "*Higher order thinking occurs when a person takes new information and information stored in memory and interrelates and/or rearranges and extends this information to achieve a purpose or find possible answers in perplexing situations*"[11] (p. 28).

The broadest attention to higher order thinking has derived from Bloom's taxonomy of educational objectives.[15-16] Bloom's taxonomy was originally formulated to help educators design assessment measures that would capture a range of ways that students engaged with material. It differentiated between questions that required higher and lower levels of cognitive engagement to complete them successfully. The levels requiring higher engagement with the content - application, analysis, synthesis, and evaluation - are considered progressively higher order thinking. This taxonomy has been widely invoked in classroom instruction and assessment design, with notions of assessment being reflected back on proposed educational reforms to better prepare students to respond to these types of "higher level" items.

However, these frameworks fall short of specifying the cognitive mechanisms that underlie higher order thinking. The utility of a definition that turns on cognitive mechanisms is that this enables the fields of cognitive science and education to align more directly to promote cross-fertilization of research and ideas aimed at facilitating these competencies in educational practice. We contend that in order for a reasoner to 'fill the gaps,' as highlighted by Bartlett or invoke productive reasoning, as Meier describes, one must conceptualize the to-be-learned information as a relational system that can be manipulated, filled, and productively refined. At the same time, cognitive science has begun to clarify the cognitive and neurological underpinnings of humans' abilities to conceptualize, compare, and mentally manipulate relationships. In particular, the literature on analogical reasoning has begun to elucidate the cognitive mechanisms underlying these skills described as higher order thinking.[17-22]

This paper articulates how analogy can serve as a cognitively grounded and generalizable definition of higher order thinking and a fundamentally different way of envisioning learning. Importantly, the cognitive specificity of this analogical model of higher order thinking lays a theoretical groundwork for connecting cognitive research with important educational questions. The next section describes a structure-mapping model of analogy (derived from Gentner[4]) that goes beyond the traditional representation of analogy as “a” is to “b” as “c is to “d” and briefly reviews the benefits and limitations of analogy in learning. Next, the way that analogy underpins expert-like thinking is discussed in three domains: mathematics, science and history. Specific pedagogical implications for this new definition of higher order thinking in each of these domains are posed.

ANALOGY AS BOTH LEARNING TOOL AND LEARNING OUTCOME

Increasingly, studies are showing that analogy and relational reasoning can produce learning in a variety of instructional contexts[23], including formal k-12 schooling of mathematics[24-26], science[27-32], history[33-37], informal museum[38], and business negotiation[39] learning contexts. Thus, analogy can be understood as a powerful learning tool. Relational reasoning also, however, can be productively understood as the goal of education. According to this view, teaching should lead students to view knowledge as something to be refined, manipulated, connected to other information, and otherwise used across contexts to serve one’s goals. Without clear guidance for how to cultivate this approach in practice, however, learning in classrooms has often been more effective at supporting memory for instructed knowledge, which is more easily assessed,[6] and in many disciplines has become an unintended but dominant focus of instruction.[1]

A Structure-Mapping Model of Analogy

Analogical reasoning has been defined[4,40-41] as a goal-oriented process of representing information and objects in the world as systems of relationships, and drawing connections across these systems of relationships. The formal, traditional way of depicting analogy is to describe a source relationship, *“a” is to “b,”* (for example “*bird is to nest”*) and finding a similar relationship within a different set of objects in a target context: *“c” is to “d”* (for example, *“bee is to hive”*). Formal analogy tasks of this type require drawing a higher order relationship, “same” between the source and target first order relationships, in this case “lives in.” Structure-mapping is the process of aligning key objects and relations within one system of relationships to another to draw higher order relationships that enable the reasoner to make inferences about the systems' commonalities, differences, or to better understand one relational system.

While the above scenario described a higher order relationship between two single relations, the source and targets can be complex relational systems as well. For example, one could conceptualize the higher order relationships between two historical battles, identifying aspects of the relationships within the two battles that are similar (e.g., they are both contexts in which disenfranchised peoples sought to overthrow a colonial government) or different (one of those attempts was successful, and another was not).

A Structure-Mapping Model of Higher Order Thinking

These well articulated cognitive mechanisms of analogy may be conceptually extended to describe the underpinnings of disciplinary higher order thinking more broadly. As will be reviewed in more detail below, many academic disciplines share the characteristic that their experts are more focused on relationships than discrete phenomena. Accordingly, directing novices and learners to greater higher order thinking requires shifting their learning aims away from memory of isolated items to aligning, comparing, contrasting, manipulating, or otherwise transforming information.

The cognition involved in higher order thinking by structure-mapping builds on the definition of analogy provided above, but can be deconstructed beyond the identification of the higher order relationship itself. Understanding the key steps of structure-mapping is important in order to develop cognitively grounded insights for supporting disciplinary higher order thinking. First, the reasoner must develop a specific way of viewing the compared representations as relational.[4-5,40] In the example above, rather than seeing the battles as lists of facts (i.e. the names of the generals, the dates and locations of the battles), the phenomenon must be represented as systems of relationships (for example relationships between the general, a colonizing nation, food scarcity, and uprising farmers).

While analogy is often described as specifically the process of generating higher order *similarity* relations (as opposed to contrastive relations), in fact these two types of comparative judgments are both engaged in any analogy.[42] In order to determine that some elements of the aligned systems are common and some are different, one must notice and attend to both similarity and contrast. For example, it is only interesting to consider the above-described battles because they have some higher order similarity (both are colonial uprisings) and some higher order contrast (only one was successful). If a person had been comparing one of these battles with a classroom math lesson, the lack of similarity between these two contexts would have made the differences uninteresting. Similarly, if one had been comparing two battles in the same war in neighbouring villages that had both been successful, the lack of difference across these cases would have made a careful comparison between the cases less useful for generating broad insights into anti-colonial revolutions. Likely, in the case of overwhelming similarity, any insights gained may have turned on *differences* between the villages (e.g., the reasoner might note that "*in spite of the smaller size of village #2, the uprising was just as successful as in village #1*").

In sum, higher order thinking, when defined as structure-mapping, is the process of representing disciplinary information as systems of relationships, aligning and comparing/ contrasting these systems to develop higher order relationships (such as same, different, or causal), and then drawing inferences, problem solving, and reasoning on the basis of those higher order relationships. This definition allows the cognitive science literature on relational reasoning to provide candidate insights into key questions within education research and practice. The following section reviews some ways that the basic research on learning by analogy may provide fruitful insights into cultivating higher order thinking in the classroom.

Learning by Analogy

The act of formulating or solving analogies has been shown to produce learning in several ways. The first derives from the literature on analogy and expertise development. In many disciplines, doing the cognitive work to organize information into relational systems is a key part of developing a more expert-like conceptualization of disciplinary information.[6,10, 23,43-44] Novices, even high knowledge/ proficient novices, tend to represent disciplinary knowledge as many discrete units, while experts are characterized by representing large bodies of knowledge as connected relational systems that can be manipulated according to situational goals. This frame for knowledge representations then has implications for organizing future learning, and provides an aim for disciplinary growth and learning.

Other work on analogy has focused on schema formation as a beneficial product of doing the cognitive work of structure-mapping. Comparing the relational structures in two cases may foster the development of a schema, or relational abstraction, of identified commonalities between two compared analogs, such that the shared relational commonality are stored as the schema, and the contextualized particulars may be discarded.[45-47] The process of aligning and mapping shared aspects of relational systems between compared representations may reveal which aspects of the relational systems are particularly central to the common underlying concept. This reasoning process may also lead to identifying key differences between the representations. Theoretically, the identified crucial commonalities or differences would then be preferentially encoded and retained by the reasoner, leading to retention for the more decontextualized relations, which could be more easily transferred and applied to new contexts and relational systems.[23,47-48]

Finally, performing structure mapping in an analogy may result in learning through re-shaping one's mental representations of one or both of the relational systems being compared.[49] Chen & Klahr found that experts tended to sometimes create source analogs to help explain a target, or they were able to retrieve a far transfer source analog, but importantly that the act of structure-mapping between the target and the source analog may enable a learning experience as well, that alters the reasoner's representations themselves.

Note that the dual roles of analogy as both learning tool and learning outcome mutually reinforce one another. Performing the work of structure mapping can give rise to more expert-like knowledge structures and schemas, which in turn potentiate more sophisticated analogies and higher order thinking.

Challenges in Learning by Analogy.

The basic research on analogy has identified several specific areas in which analogy may break down and preclude successful learning outcomes, which may have useful practice implications for educational environments designed to encourage higher order thinking. Presenting participants with opportunities for relational structure mapping can enhance learning, solution generation, and abstraction, but the benefits are not guaranteed.[50-52] Two primary challenges that emerge from the experimental literature include 1) learners may not recognize the possibility or utility of making an analogy despite there being available relational correspondences, and 2) learners may be hindered by the high processing demands of such higher order thinking.

When participants identify that they should be making an analogy, they often appear competent and able to both draw and benefit from the key higher order structure mappings. However, young children or domain novices, in particular, tend to notice correspondences between sets, problems, or concepts based on object properties (e.g., like the appearance of a triangle) versus the relationships within those representations (e.g., the relations between angles and line segments within a polygon).[48, 50] Thus, in a classroom, students may be more likely to relate situations or problems on the basis of superficial, non-essential features, rather than on their underlying principles. On the other hand, teachers, who have more expertise, may provide instruction that presupposes the fundamental, relational commonalities will be apparent to their students, and that students will be able to generalize (analogically) to new situations.[43]

This challenge is compounded by children's limited processing capacity. Representing information as integrated relational systems and then aligning, mapping, and drawing inferences based on these systems, all require working memory and inhibitory control resources.[18-22] Partly this is because all phenomena contain many features, some based on appearance (object features) and some based on relational correspondences, and constructing an analogy requires disattending to features of the source and target phenomena that are irrelevant to the pragmatic goal of the analogy.[5, 53-54] Simultaneously, conceptualizing relations, and integrating multiple relations into a more complex relational system requires working memory.[55] Both children and adults fail to reason relationally when under high working memory load [18-21,56], when inhibitory control demands are high,[54] when under stress,[57] or when their knowledge in a domain is limited [58-61].

While the ability to handle complex relational systems and inhibit attention to objects improves with age, [55, 62-63] individual differences in executive function, knowledge, and experience are also related to children's analogical aptitude. As shown in Figure 1, in a large-scale longitudinal study, children's early inhibitory control and knowledge (vocabulary) at school entry predicted analogy skill at age 15, even when controlling for analogy skill at grade 3.[55] This longitudinal relationship provides further evidence that children's ability to perform analogical, higher order thinking will increase over time and with resource capacity.

Successful classroom analogies, therefore, will be ones presented in such a way that learners are attentive to the need for doing higher order thinking and feel pragmatically compelled to engage in the effort to do so. Further, learning from these interactions will only happen if the reasoner has adequate processing resources to disattend to the object properties of compared phenomena by engaging cognitive control, and instead represent the compared phenomena as integrated relations, and align/map/ and draw inferences based on higher order relationships in working memory. As in the assistance dilemma[64], this requires a balance between ensuring that the task is adequately challenging that the learner must engage his/her analogical reasoning, but not overly taxing of his/her cognitive resources.

A meta-analysis of contrasting case research clarified that despite the challenges of meeting students' needs appropriately, comparing representations led to on average greater learning than other activities.[23] Specific strategies that were particularly effective across the sample of published studies included asking learners to find similarities between cases, providing principles after the comparisons, using perceptual content, and testing learners immediately are all associated with greater learning.

These ideas as well as pedagogical strategies for reducing inhibitory control and working memory demands during instructional opportunities for higher order thinking are discussed in the context of the three disciplines below. The utility of defining higher order thinking as structure-mapping is illustrated in the contexts of Mathematics, Science, and History Education, and specific ways that the literature on analogy may inform disciplinary research are highlighted.

Analogy in Mathematics Education

Mathematical expertise has been described explicitly in terms of analogy, as by Polya[7] who famously stated that mathematicians draw analogies, and expert mathematicians draw analogies between analogies. At the same time, reasoning skill within school mathematics is an area of serious underperformance by US students,[8,35,65-67], and in particular, these students are showing a lack of understanding that mathematics is a flexible system of relations that can be manipulated and considered. A survey including problem solving and interview methods administered to high school graduates in the United States who have enrolled in community colleges, for example, reveals with striking clarity that many students do not conceptualize mathematics as a reasoning discipline, but rather as a set of procedures to be memorized.[65-66] This emerged in this sample both from explicit statements to this effect, such as "In math, sometimes you have to just accept that that's the way it is and there's no reason behind it" (p. 8) as well as implicitly in their approach to problems. Students almost universally attempted to retrieve procedures to solve each problem, failing to notice relationships between the problems or concepts (e.g., failure to recognize that that $\frac{1}{3}$ is the same as 1 divided by 3), or between problems that could ease the problem solution procedures. For example, Figure 2 reveals the way that two students solved a series of related problems. Despite the possibility for them to draw relationships between these problems that would greatly reduce the calculation demands of the problems, students did not notice the utility of using one solution to help solve the next, nor did they notice the discrepancies between the answers to the sequential problems.

Clarity in Defining Aims for Higher Order Thinking in Mathematics

These students' lack of attention to potential analogies and relational correspondences may be related to teaching practices. Part of the challenge for educational reform in this area may lie in the lack of clear language for describing the aims for mathematics instruction. In a classic paper, Skemp[26] described the problem of a "*faux amis*" regarding the term mathematical "understanding." Faux amis is a french term to describe words that appear to be homonyms across languages, but in fact have distinct meaning. For example the word "chef" in French means "head of business," not necessarily chief cook, though an English speaker might be quite confident that s/he understands the meaning of the term to be head of a kitchen. Skemp suggests that the term *understanding* similarly has two meanings when applied to classroom mathematics, such that teachers and researchers who agree that they both seek to increase students' mathematics understanding may actually be relying upon different definitions. He distinguishes between "relational understanding," as knowing what to do and why, and "instrumental understanding," which is retaining and utilizing mathematical rules appropriately. Despite appearing quite different as described in theory, in practice the distinction between these two forms of understanding is less clear. When a teacher who knows mathematics as a set of rules seeks to teach students mathematics with understanding, evidence of student understanding is apparent through students' ability to reproduce the rules as taught on problems that are similar to the instructed problems. Students' inability to recognize commonalities between conceptually similar problems that appear different in some ways was listed by U.S. algebra teachers as a top challenge for their students.[8]

Defining higher order thinking as relational structure mapping is a productive way to clarify the distinction between instrumental and relational understanding that may make the faux amis a less problematic contrast. While attempts to make learning in classroom mathematics more conceptual have been difficult to articulate and implement,[8, 67] providing a cognitively grounded model of higher order thinking that pushes teachers to consider mathematics as crucially about teaching students to see mathematics as sets of relationships between problems, solution strategies, and concepts, may be effective.

Mathematics Classroom Analogy Practices

Analyses of classroom mathematics instruction have revealed that analogy is a common part of teacher practices in many countries, including the United States, China (Hong Kong,) and Japan. [68-70] At the same time, there are key differences in how these countries' teachers provide pedagogical cues to focus students' attention to key higher order relationships of interest, rather than allowing them to focus primarily on the object-features of the representations. In parallel to achievement patterns, U.S. teachers are much less likely to provide these cues than teachers in higher achieving regions – Japan and Hong Kong.

Figure 3 reveals the differences in frequencies within which teachers in these three regions used a range of typical, everyday teaching behaviors that could support students in relational reasoning. There were no reliable differences in numbers of analogies made across the sample, though there were variations in the frequency within which these analogies were produced with a visual-spatial representation, or that two compared visual representations were available simultaneously, factors that may lessen the high cognitive load imposed by analogies. There were also systematic variations in the visual alignment between these representations (e.g., between two problems written on the chalkboard), spatial organization, animation, and imagery.

An additional difference of interest was in the frequency with which teachers used hand and body movements to link between the compared representations.[71-73] Linking gestures can be powerful tools to draw learners' attention to key relational comparisons across relational systems,[71] and teachers in the U.S. and regions that outperform U.S. students – Hong Kong and Japan, use them frequently, which means this could be a feasible way to help students attend to key relationships and disattend to object features.[73] While all teachers examined used high rates of gesture, however, the cross-cultural analysis introduced above revealed that most teachers used gestures that referenced only one representation or a second, but did not link between these representations. Some teachers did use linking gestures, and this correlated with achievement patterns at a country-wide level, such that teachers in higher achievement countries used linking gestures significantly more frequently when compared with the proportion of analogies containing linking gestures in the U.S. sample.[69]

Strategies for Encouraging Higher Order Thinking

Several lines of experimental research indicate that everyday ways of reducing students' processing load during instructional analogies may not only be correlated with high achievement, but also may be causally related to improved student learning and relational reasoning. Richland and McDonough[74] manipulated the presence or absence of a combination of these practices during a videotaped lesson comparing solutions to a permutation and a combination problem. The same problems were taught in two lessons using the same instruction, but in one version the problems were presented sequentially and not visible simultaneously, while in the second version, visual-spatial representations of the problems were visible simultaneously and linking gestures were used to move between the spatially aligned representations. Posttest data revealed that participants in both conditions demonstrated learning for the instructed procedures, but participants in the latter, high support for alignment condition, were better able to solve problems that required more structural attention to the core ideas, while the sequential condition supported attention to object features. Thus these participants appeared to have been more successful at analogical reasoning during instruction, and it appears that their learning for the two instructed strategies was in turn more schematic and generalizable.

Rittle-Johnson and Star[24-25] have similarly shown that simultaneous visual-spatial representations of analogs can support opportunities for comparison within classroom mathematics instruction, and such supported analogies can enhance students' procedural and conceptual understanding. In these studies, students were provided worksheets in which they either saw problems solved two ways presented simultaneously, or problems solved in one way at a time, with the two solutions presented sequentially on different pages of a packet. In both cases an astute student would have drawn the relationship between these analogs, but the former version placed more explicit attention to the comparison and made the relations easier to identify. The students discussed these problem solutions in collaborative pairs based on discussion prompts. As shown in Figure 4, being shown two representations simultaneously and being explicitly asked to compare them in these groups led to greater retention for the taught procedures, as well as more transfer to novel problems.

Interestingly, while these studies revealed that the simultaneously visible solutions (high support for analogy) condition was helpful in promoting flexible, conceptual learning, students' prior knowledge also appears to be a determining factor to explain when an analogy will be effective.[75] Students needed to have adequate prior knowledge in order to benefit from the opportunity for higher order thinking. In this study, students without any intuitions about algebraic problem solving on a pretest benefited from sequentially attending to two separate solution strategies, while the opposite was true for students with even rudimentary intuitions about doing algebra. This suggests that prior knowledge for the compared representations must be adequate to support reasoning by analogy. In classrooms, this is an important constraint, and one that teachers must be aware of in order to best ensure students are supported in learning from instructional analogies.

Current studies are underway to investigate the import of these strategies when used independently, including examinations of the role of visual-spatial representations, spatially aligned visual representations, and linking gestures, to evaluate if certain strategies are particularly helpful or important to promoting students' analogical/ relational thinking.

Overall, the converging evidence suggests that engaging students in higher order thinking in mathematic requires providing pedagogical support for drawing their attention to problems and concepts as relational representations that may be aligned, connected, compared, contrasted, integrated and refined. These supports may include making the relations between representations highly explicit, using hand and body movements to link mathematical representations, using visual representations, making visual representations of compared ideas visible simultaneously, and making those representations spatially aligned while visible. Despite converging evidence that these are important, teachers often inadequately support their students in re-representing mathematical objects (problems or concepts) as relational representations, and ensuring that they attend to the similarities and differences between these representations. This may contribute to the finding that many students graduate k-12 schooling without a conceptualization of mathematics as a field for reasoning about relationships, instead, viewing mathematics as a discipline based on memorization for procedures as static objects.

ANALOGY IN SCIENCE EDUCATION

As in mathematics, analogy plays a privileged role as a cognitive mechanism underlying school science. Two key ways that analogy functions in classroom science include, first, as a way for students to draw relationships between target scientific phenomena and more easily represented lab materials, visual representations, models, or well understood scientific phenomena. Second, a key element of scientific thinking involves understanding the natural and human-built world as complex systems of relationships that may be further compared, contrasted, integrated, or otherwise explained. For example, one can view an ocean as a set of principles to recall (name, % of the world's surface, number of species, depth, tides, etc), or one can represent it as a highly interconnected, complex ecological system [6,9,76-77]. Considering both roles of relational reasoning will help to improve students' disciplinary higher order thinking within science.

The National Research Council released a framework for k-12 science[9], followed by standards for science instruction[78], that together emphasize the crucial nature of relational thinking across science fields. A foundational part of the framework was to develop cross-cutting themes that were described as having application across all areas of sciences, and in particular were designed to serve as links across areas of science. When one examines these carefully, one realizes that all of these themes specifically rely on relational thinking. The cross-cutting themes include: Patterns, similarity, and diversity; Cause and effect; Scale, proportion and quantity; Systems and system models; Energy and matter; Structure and function; Stability and change. All of these themes involve attending to and identifying recurring patterns evident through similarities, contrasting cases, number, scale or quantity. The types of relationship patterns also may be categorized, including as demonstrating causality, positive or negative relationships, stability despite contextual changes, or predictable or unpredictable change.

The framework also specifically highlights that making these themes explicit and integral to both practice and content acquisition will help learners develop a structured, coherent representational system of scientific inquiry. As recognized in the cognitive science literature, relational thinking and abstractions are often conceptualized to emerge naturally and easily from learning objects and individual representations, however the experimental literature finds this is often not the case, particularly for domain novices.[43,79] For this reason, as in mathematics, using pedagogical moves to draw explicit attention to key relationships and making this a crucial part of instruction may be key to ensuring that students engage in disciplinary higher order thinking in science.

Thus, we can extend the definition of higher order thinking posited above to develop a cognitively grounded definition that pertains to science and draws pedagogical attention to the core relational thinking that cross-cuts all areas of scientific knowledge. This is not intended to replace definitions or theories of scientific reasoning[78] or conceptual change,[79] but rather to provide a specific meaning for higher order thinking that builds on the cognition of relational reasoning and can be applied across academic domains.

Studies of everyday use of science analogy has revealed that analogies are regularly used by people engaging with science at many levels, from k-12 classroom teaching and learning,[23,80] to nobel prize winning biological science laboratories.[79,81-82] Among experts in the science laboratory, analogy is part of everyday scientific discourse and training. Analogical thinking or discourse has been identified as playing a role in major creative insights, as part of communication between scientists or with non-experts, as a strategy for making sense of discrepant results, or as a means for solving unexpected problems.

In the K-12 student context, analogies are regularly used in both textbooks and classroom discourse,[79] though there is converging evidence that acquisition of complex, integrated relational systems of knowledge within science cannot happen without explicit instructional support. [28-31] This may take many forms, including direct instruction[31], scripted analogy activities,[29, 85] or carefully designed technologies for promoting knowledge integration[86-87], but as in the domain of mathematics, it is unlikely that learners will notice and engage in higher order thinking without the explicit direction regarding how representations should be aligned and compared.

One specialized role of higher order thinking in science instruction is to serve as a cognitive mechanism underlying conceptual change. Much research has demonstrated that analogical reasoning can be a mechanism of conceptual change, enabling learners to move from a naive representation of a scientific phenomenon to a more expert-like representation.[41, 88-90] Students' preconceptions may be in line with accepted scientific theory, also called "anchoring conceptions"[85] or may be in contrast to accepted scientific theory as misconceptions.[92] Either way, conceptual change requires relational reasoning as new information is assimilated into the existing relational structure, or the structure is modified, expanded, integrated with other relational representations, or otherwise refined.[90-91]

Distinguishing among several hypothesized models for conceptual change is beyond the scope of the current paper, but all take as a premise that people do not learn new scientific information by encoding and retaining discrete, atomistic information, but rather new data are processed by evaluating alignment with reasoners' existing relational structures, which are then refined, integrated with, or imposed on the new information. As such, analogical reasoning functions as a cognitive underpinning of conceptual change.[90]

Strategies for supporting Analogical Learning in Science. As in the mathematics section above, analogies are frequently used for instruction in science, but merely invoking an instructional analogy is not adequate to produce higher order thinking by students. Students may fail to notice the utility of engaging in relationally representing the scientific phenomenon being compared, or fail to notice the relevance of one representation to help understand another. A common yet little appreciated failure is that students who engage with a diagram, model, or science laboratory materials in the classroom may fail to notice the correspondences to the scientific phenomena they are intended to model.[9,86,76,89-91] This may be because the model and the natural phenomenon have different object properties (e.g., a diagram of the solar system with concentric orbit circles versus the physical objects within the solar system, which cannot be viewed simultaneously and which do not have visible orbit pathways.

A student may become proficient in interacting with the visualization, but without explicit support for aligning and mapping between this representation and the natural phenomenon, any mental representation of the diagram may be encoded simply as a discrete object.[9,76] One key part of teaching higher order thinking in science, therefore, requires not only supporting students in manipulating and engaging with scientific models, but also explicitly supporting them in connecting these models with aligned representations of the scientific phenomena.[86]

Several frameworks for how analogy can be used to support learning of science have been proposed. One model is to use bridging analogies: analogies between well-known entities and more novel scientific phenomena.[45] These bridging analogies provide learners with a platform from which to develop inferences and to prompt conceptual change, moving from one's original ideas about a target phenomenon to reformulate them based on comparison with the source. Importantly, however, as is a recurrent theme, this process can be an effective model for promoting conceptual change, but bridging analogies may require very explicit support and pedagogy by the instructor in order to be useful to learners.[29]

Another framework is to provide instructors or students with an explicit set of steps that must be completed in order to ensure that an analogy is used most productively, with reduced chance for encouraging misconceptions. Such steps are described in the Teaching With Analogies Model[32] which is a model for instructional analogies deriving from successful teachers and textbooks. The model includes the following six steps: (1) Introduce the target concept, (2) Review the analog concept, (3) Identify relevant features of the target and analog, (4) Map similarities, (5) Indicate where the analogy breaks down, and (6) Draw conclusions.

Overall, this brief review highlights ways that analogies function as an integral part of science classroom instruction. The definition of higher order thinking as relational structure-mapping applies well to these common, central parts of science instruction, and helps to provide an aim for teaching students an expert-like way of engaging in scientific thinking. Specifically, higher order structure-mapping is revealed to be integral to experimentation, model-use, and conceptual change in science. Ensuring that students benefit from opportunities for such higher order thinking, however, requires direct instruction to attend to relational structure. Further, pedagogical practices to draw students' attention to higher order similarities as well as their limits are necessary, and can include visual/spatial representations and simultaneous presentation of these models, use of dynamic technology for knowledge integration and interaction with visualizations, or teaching that meticulously guides students in bridging from prior knowledge representations to more expert-like knowledge while revealing limits to naïve analogs.

ANALOGY IN HISTORY EDUCATION

A much smaller yet thought-provoking literature on the cognition of history education suggests that as in mathematics and science, relational structure mapping plays a key role in the learning of history. Very much like in mathematics and history, analogy plays both an explicit role as a pedagogical tool for learning history[92], and it serves as a cognitive underpinning for the reasoning about relationships that characterizes expertlike historical thinking, through argumentation, constructions of causal and other higher order relationships, and similarity/ contrast goals[93].

Critical theoretical treatments of the constructive role of history are drawing attention to the problematic nature of construing history as a list of canonical facts that should be memorized by well-educated members of society.[10, 34] Beyond the problematic nature of the selection process determining which facts should be part of the educational cannon lies a greater challenge based on contrasting epistemologies of what historical thinking should entail.[34-36] An alternative to the canonical fact epistemology about the nature of history is that learning history entails learning to understand patterns of human experience as situated within its larger temporal context.[33, 94] This has led to deep and rancorous debates about standards for history education,[10] and subscription to these two models seems to vary internationally and between teachers.[93,95]

In particular, there is a hypothesized disjunction between the way novices and historians conceptualize the doing of history, and consequently, what education should attempt to promote. Wineburg[10, 96] described studies comparing historians (experts, without deep knowledge of the period in the task) and students or k-12 teachers (novices or proficient novices). He finds that when provided with primary documents, even historians who know little of the time period engaged in deep relational thinking, seeking to embed any interpretation of the primary documents within a larger system of relationships within the historical period. These readers asked questions of themselves and the data, engage in lengthy processes of comparing and contrasting across these primary documents themselves, and note assumptions they make and the necessity of reviewing them based on the primary documents and additional materials. Thus as in mathematics and science, the experts were constructing relational systems and manipulating, aligning, revising, and drawing inferences based on these relations. In contrast, the novices or proficient novices who often knew more facts about the period, interpreted the documents as discrete objects, without the attention to the larger relational system. These participants were more likely form judgments based on their own intuitions or interpretations of single documents (e.g., whether a statement portrayed racism) and made less cautious interpretations of the data.

Importantly, however, as noted by Wineburg,[10] many of the activities performed by these novices involve productive thought, and would be classified as being in the higher levels of Blooms taxonomy. These are the types of practices students are encouraged to perform during periods of instruction that are seeking to encourage the students' conceptual thinking. At the same time, in history, these can be irresponsible activities without also maintaining larger system-wide alignments and mappings between the relationships identified in the primary documents and the larger social-historical context. Thus the goal orientation element of analogical structure mapping is shown to be crucial here. Simply allowing structure-mapping to unfold is not necessarily executing effective higher order thinking, but executing this reasoning process in a pragmatically sensitive, systematic way that critiques the validity of mappings is essential.

The National Council for History Education (http://www.nche.net) frames this as fostering "History's Habits of Mind" in learners, distinguishing learning modes of framing and treating historical and current phenomena as systems of relations from knowledge of historical facts or accounts. Thus, this approach suggests that historical cognition requires moving beyond memorization of facts to developing representations of history as connected systems of relationships that can be compared, integrated further, contrasted, or otherwise manipulated. Thus the structure-mapping model of higher order thinking should be useful to articulating instructional aims in history as well.

Another role of analogy in history education that serves a complicated role is the process of drawing higher order relations between the reasoner him/herself and an individual or phenomena in a different historical period.[24,25,97] The early 20th century educational psychologist, Charles Hubbard Judd[98] foreshadowed this model of higher order thinking with the observation that learners bring their own lens of time and place to bear on representations of history, and must learn to be attentive to these relationships. This practice must be critically approached, however, since the application of assumptions and expectations based on one's current complex relational context cannot be uncritically imposed on the other historical period.[10] Wineburg suggests that "presentism," or the established modes of thinking in one's everyday experience imposes an extremely high burden on historical thinking, which must attempt to determine and interrogate these established patterns before they can influence interpretation of history.

He illustrates the challenges of using analogies in history though an example of a 21st century student learning about the Battle of Lexington. This powerful example also illustrates how the conceptualization of higher order thinking as relational reasoning provides a more expert-like aim for instruction than a definition deriving from Bloom's taxonomy. The student read a series of primary sources about the Battle and went beyond memory for the documents or the facts of the battle by getting swept into the drama of the event, made evaluations about the outcome of the battle and the participants' strategies, constructed a causal argument for what happened and why, and applied his knowledge of the documents to predict the configuration of the battle field. These all could have been used to show that this student was engaging in high quality historical thought. On the other hand, he selected a picture to represent the battle that would have been highly sensible based on modern notions of warfare but that would have been widely shunned at the time period for a lack of honor and breach of standard practice, hiding behind walls. Thus this student allowed his conceptualization of warfare situated in his existing worldview to impose constraints on his interpretation of the historical facts.

The Cognitive Science of Higher Order Thinking in History

To impose a cognitive lens on this process, then, one might determine that higher order thinking in history requires that one make explicit the system of contextual relationships within which one's experiences are embedded, and seek to develop the same type of richly structured system of relationships for the historical period. Subsequently, these two relational representations can be aligned and higher order relationships drawn – such as similarity, difference, and inferences may be drawn to explain the processes.

Processing Capacity

Bringing the lens of analogical thinking and higher order structure-mapping to bear on the context of history education allows for the introduction of novel ideas about the role of cognitive processing demands on the learning of history. Since learners under working memory load have fewer resources to deploy for relational integration, alignment and structure-mapping, pedagogical practices that place high emphasis on historical fact retrieval may unwittingly further decrease the likelihood that these students will integrate and processes the instructed historical focus in an expert-like way.

Similarly, students under working memory load will have reduced capacity for inhibitory control. Importantly, while drawing higher order structure mappings is the first step, the student must also inhibit the assumptions s/he makes based on these mappings. Analogies to one's contemporary time and place for example are powerful, but students will likely engage in unproductive structure-mapping that results in misconceptions, so assumptions must be held and considered explicitly until they are weighed and determined to hold. Thus drawing higher order relations must happen concurrently with inhibitory control of unconsidered assumptions.

They key to instruction, therefore, may lie in ensuring that pedagogical practices of supporting explicit, critically considered analogy are adequate to provide learners with direction *and* processing resources available to make the intended structure-mappings (and not make unintended ones). Therefore, strategies that have been experimentally demonstrated to support reasoned relational thinking in mathematics and science may be usefully deployed in history.

These strategies include using visual representations of key relationships and making representations of compared systems visible simultaneously. So, one might, for example, use representations within the historical period to contrast with aligned representations of the present period to alleviate attention to facts that will typically overwhelm a child's working memory resources. Comparing the photographs used as lures in the Battle of Lexington example, for instance, might productively illuminate the systems of relationships at work in students' current experience versus during the temporal period of the battle. This comparison process might be additionally supported by linking gestures that move between visual representations such as photographs or even diagrams, and spatial alignments between key visual/spatial representations may also provide this support.

Further, cognitive scientific treatments of analogy suggest that very explicit prompts to facilitate intended structural alignments are necessary to ensure that learners notice the utility of drawing higher order relationships. While a teacher may imagine that learners notice how differently relationships unfold across historical periods, or differently from students' own contemporary experiences, this may not emerge without direct instructional attention. In history, these prompts could take the form of activities and assignments that explicitly require learners to construct a representation of the key relationships and contextual influences within the historical period of interest, situating key facts into the larger contextual framework. Thus drawing inferences, making connections, and other higher level cognitive tasks performed with these materials would need to be situated within the larger framework of seeing history as systems of relationships that are mapped together, and in which students are always engaged in a process of conceptual change, bringing their own analogs to bear on prior periods.

CONCLUSION

In conclusion, the key to teaching higher order thinking across mathematics, science, and history is to conceptualize learning as developing and manipulating relational systems. Such a definition will help in all three fields to clarify what constitutes expert-like thinking in that discipline, and what would be high quality instructional activities that lead to successful attainment of this goal. In some ways this is a fundamental restructuring of the way learning is conceptualized in k-12 classrooms, moving from a cognitive model of learning that derives from memory processing to a cognitive model that derives from analogical reasoning.

Aligning these instructional goals for higher order thinking with the cognitive literature on analogy also provides insights into the role of cognitive and developmental processing constraints on children's relational thinking. This awareness can help instructors recognize their students' need for very explicit support to notice and draw meaningfully on higher order relationships. Since analogy is already an oft-used practice within all three of these disciplines, better theory for how to optimize these opportunities for higher order thinking will support teachers in supporting their students' disciplinary skill development.

Notes

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FIGURE CAPTIONS

Figure 1. Estimated verbal analogy scores at 15 for children with low[a], average and high vocabulary and EF skills at entry to school*. (Reprinted [52])*

Figure 2. Community college students' solutions to a series of problems reveals attempts to execute procedures without attention to simplifying relationships between problems (reprinted from Givven et al).[74]

Figure 3. Frequency of supports for analogy across 8th grade mathematics classrooms within nations with high and lower national achievement in mathematics (Reprinted[77])

Figure 4. Posttest accuracy on problems familiar or novel to participants, for students who compared two simultaneously visible solutions to a problem versus viewed these solutions sequentially. (Reprinted, p. 569[24])

Tables

[Please insert any tables here]

Further Reading/Resources

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